Lecture 13 Part 2 Digital Pulse Amplitude Modulation

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Review of Previous Lecture

- Digital communications
  - Group bits into symbols of J bits
  - Map symbol to unique amplitude
  - Want receiver to have fewest number of errors
  - Transmitter and receiver decide the # bits/symbol (constellation size or J) and hence the bit rate J fsym
- Quadrature amplitude modulation (QAM) is two-dimensional PAM; i.e., QAM generates two baseband PAM signals, upconverts each, and then subtracts them.
- Cellular connection
  - User equipment (phone) requests a connection with a base station
  - Base station sends a training signal to the user equipment (e.g. pseudo-noise)
  - User equipment applies signal processing methods to increase the SNR of the received signal
  - User equipment sends four-bit unsigned channel quality index (CQI) to the base station to indicate the received SNR (e.g, 0 = -5 dB, 1 = -3 dB, 2 = -1 dB, etc.)
  - The basestation will decide on how many bits/symbol bases on the reported SNR
  - The user equipment will also send a training signal to the base station, the bas station will apply signal processing methods to increase the SNR of the received signal, make a decision on how bits/symbol the user equipment will use when transmitting bits, and sends the bit allocation to the user equipment.

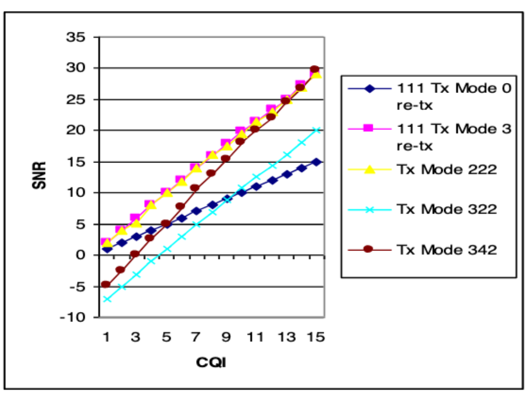


Fig. 6. SNR vs CQI for different Tx modes

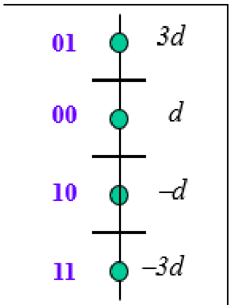
Table 7.2.3-1: 4-bit CQI Table

CQI index	modulation	code rate x 1024	efficiency
0	out of range		
1	QPSK	78	0.1523
2	QPSK	120	0.2344
3	QPSK	193	0.3770
4	QPSK	308	0.6016
5	QPSK	449	0.8770
6	QPSK	602	1.1758
7	16QAM	378	1.4766
8	16QAM	490	1.9141
9	16QAM	616	2.4063
10	64QAM	466	2.7305
11	64QAM	567	3.3223
12	64QAM	666	3.9023
13	64QAM	772	4.5234
14	64QAM	873	5.1152
15	64QAM	948	5.5547

https://ytd2525.wordpress.com/2014/02/02/cqi-channel-quality-indicator/

Gray Coding

- Adjacent symbols only differ by 1 bit
- If receiver gets wrong symbol, it is likely an adjacent symbol and only 1 bit is wrong
- Likely can be corrected using error correcting codes
- PAM constellations can always be Gray coded, e.g.



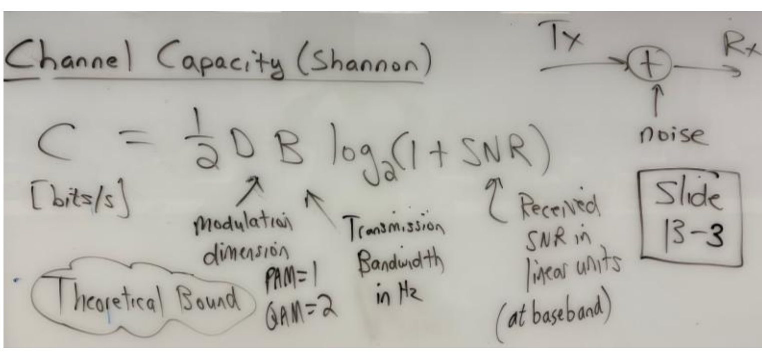
4-level PAM Constellation Map

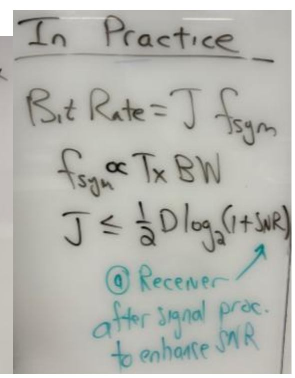
Symbol Amplitude to Baseband Waveform

- Interpolate (upsample by L and then apply pulse shaping filtering) to convert the discrete-time signal of symbol amplitudes sampled at fsym to a signal sampled at fs
- D/A converter (sampling at fs) produces an analog continuous-time baseband signal

Channel Capacity

- Maximum possible bits/second with ideal channel and only additive white gaussian noise
- Theoretical upper bound





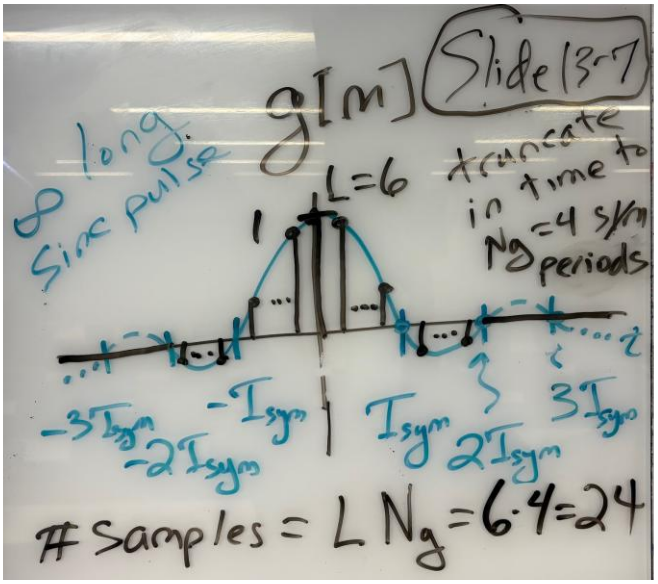
PAM Transmission

- 2-PAM
  - Symbol amplitudes are d and -d
- 4-PAM
  - Symbol amplitudes are -3d, -d, d, 3d
  - 4 amplitudes spaced 2d apart and are centered at 0
- d is a constant (voltage) value determined by the transmitter
  - Can be different in different systems, so we generalize its value when looking at constellations

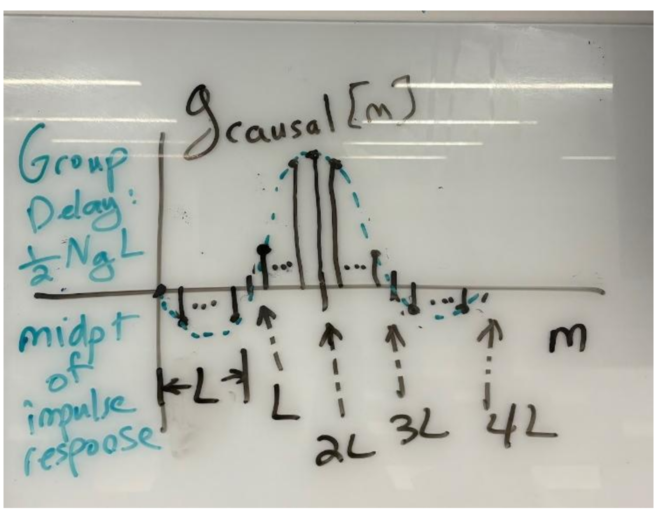
- When upsampling by L, pulse shaping, and converting to amplitude, should not change symbol amplitudes
    - Receiver expects certain amplitudes for symbols
    - Pulse shaping filters must meet two constraints
        - Frequency domain: lowfass filter with cutoff frequency pi/L
        - TIme domain: impulse must be carefully designed to not change symbol amplitudes (every Lth sample must be zero except the middle sample)

<u>Pulse Shaping Filter Example</u>

- Sample continuous time sinc for 4 symbol periods



- Not causal, contains negative discrete-time indices
    - Think, cannot use future to calculate the current output of a filter
- Shift to right by 2L samples
    - Filter impulse response goes from indices 0 to 4L
    - Peak amplitude value occurs at index 2L
    - Group delay of 2L



Polyphase Filterbank

- When computing output of pulse shaping filter from upsampled filter, have L-1 0s for each symbol, so many wasted multiply and add operations
- Solution: polyphase filterbank
- Compute only non-zero multiplies for each output in parallel
- Output serial words (amplitudes) using a commutator
- Filters are running at a rate of f_{sym} not Lf_{sym}
- Factor of L savings in number of multiplications without changing signal quality – so we implement the filter bank in practice over the upsample and filter approach